1. The dimension of the space spanned by the vectors $(-1, 0, 1, 2)$, $(-2, -1, 0, 1), (-3, 2, 0, 1)$ and $(0, 0, -1, 1)$ is

   A. 1
   B. 2
   C. 3
   D. 4.

2. How many onto functions are there from a set A with $m > 2$ elements to a set B with 2 elements?

   A. 2^m
   B. $2^m - 1$
   C. $2^{m-1} - 2$
   D. $2^m - 2$.

3. The function $f : \mathbb{R}^2_+ \to \mathbb{R}$ given by $f(x, y) = xy$ is

   A. quasiconcave and concave
   B. concave but not quasiconcave
   C. quasiconcave but not concave
   D. none of the above.

4. The function $f : \mathbb{R}^2_+ \to \mathbb{R}$ given by $f(x, y) = xy$ is

   A. homogeneous of degree 0
   B. homogeneous of degree 1
   C. homogeneous of degree 2
   D. not homothetic.

5. You have n observations on rainfall in centimeters (cm) at a certain location, denoted by x, and you calculate the standard deviation, variance, and coefficient of variation (CV). Now, if instead, you were given the same observations measured in millimeters (mm), then

A. the standard deviation and CV would increase by a factor of 10, and the variance by a factor of 100

B. the standard deviation would increase by a factor of 10, the variance by a factor of 100, and the CV would be unchanged

C. the standard deviation would increase by a factor of 10, and the variance and CV by a factor of 100

D. none of the above.

6. You have n observations on rainfall in centimeters (cm) at two locations, denoted by x and y respectively, and you calculate the covariance, correlation coefficient r, and the slope coefficient b of the regression of y on x. Now, if instead, you were given the same observations measured in millimeters (mm), then

   A. the covariance would increase by a factor of 10, b by a factor of 100, and r would be unchanged

   B. the covariance and b would increase by a factor of 100, and r would be unchanged

   C. the covariance would increase by a factor of 100, and b and r would be unchanged

   D. none of the above.

7. Let $0 < p < 100$. Any solution (x^*, y^*) of the constrained maximization problem

$$\max_{x,y} \left(\frac{-1}{x} + y \right)$$

subject to

$$\begin{aligned} px + y &\leq 10, \\ x, y &\geq 0, \end{aligned}$$

must satisfy

   A. $y^* = 10 - p$

   B. $x^* = 10/p$

   C. $x^* = 1/\sqrt{p}$

D. none of the above.

8. Suppose the matrix equation $Ax = b$ has no solution, where A is a 3×3 non-zero matrix of real numbers and b is an 3×1 vector of real numbers. Then,

   A. The set of vectors x for which $Ax = 0$ is a plane.

   B. The set of vectors x for which $Ax = 0$ is a line.

   C. The rank of A is 3.

   D. $Ax = 0$ has a non-zero solution.

9. k people get off a plane and walk into a hall where they are assigned to at most n queues. The number of ways in which this can be done is

   A. C_k^n

   B. P_k^n

   C. $n^k k!$

   D. $n(n+1)\ldots(n+k-1)$.

10. If $Pr(A) = Pr(B) = p$, then $Pr(A \cap B)$ must be

    A. greater than p^2

    B. equal to p^2

    C. less than or equal to p^2

    D. none of the above.

11. If $Pr(A^c) = \alpha$ and $Pr(B^c) = \beta$, (where A^c denotes the event 'not A'), then $Pr(A \cap B)$ must be

    A. $1 - \alpha\beta$,

    B. $(1 - \alpha)(1 - \beta)$

    C. greater than or equal to $1 - \alpha - \beta$

    D. none of the above.

12. The density function of a normal distribution with mean μ and standard deviation σ has inflection points at

    A. μ

    B. $\mu - \sigma, \mu + \sigma$

C. $\mu - 2\sigma, \mu + 2\sigma$

D. nowhere.

13. In how many ways can five objects be placed in a row if two of them cannot be placed next to each other?

    A. 36

    B. 60

    C. 72

    D. 24.

14. Suppose $x = 0$ is the only solution to the matrix equation $Ax = 0$ where A is $m \times n$, x is $n \times 1$, and 0 is $m \times 1$. Then, of the two statements (i) The rank of A is n, and (ii) $m \geq n$,

    A. Only (i) must be true

    B. Only (ii) must be true

    C. Both (i) and (ii) must be true

    D. Neither (i) nor (ii) has to be true.

15. Mr A is selling raffle tickets which cost 1 rupee per ticket. In the queue for tickets, there are n people. One of them has only a 2-rupee coin while all the rest have 1-rupee coins. Each person in the queue wants to buy exactly one ticket and each arrangement in the queue is equally likely to occur. Initially, Mr A has no coins and enough tickets for everyone in the queue. He stops selling tickets as soon as he is unable to give the required change. The probability that he can sell tickets to all people in the queue is:

    A. $\frac{n-2}{n}$

    B. $\frac{1}{n}$

    C. $\frac{n-1}{n}$.

    D. $\frac{n-1}{n+1}$.

16. Out of 800 families with five children each, how many families would you expect to have either 2 or 3 boys? Assume equal probabilities for boys and girls.

    A. 400

    B. 450

C. 500

D. 550

17. The function $f : \mathbb{R} \to \mathbb{R}$ given by

$$f(x) = \begin{cases} \frac{x}{|x|}, & \text{if } x \neq 0, \\ 1, & \text{if } x = 0. \end{cases}$$

is

A. concave

B. convex

C. neither concave nor convex

D. both concave and convex

18. As $n \to \infty$, the sequence $\{\frac{n^2+1}{2n^2+3}\}$

A. diverges

B. converges to 1/3

C. converges to 1/2

D. neither converges nor diverges.

19. The function $x^{1/3}$ is

A. differentiable at $x = 0$

B. continuous at $x = 0$

C. concave

D. none of the above.

20. The function $\sin(\log x)$, where $x > 0$

A. is increasing

B. is bounded and converges to a real number as $x \to \infty$

C. is bounded but does not converge as $x \to \infty$

D. none of the above.

21. For any two functions $f_1 : [0,1] \to \mathbb{R}$ and $f_2 : [0,1] \to \mathbb{R}$, define the function $g : [0,1] \to \mathbb{R}$ as $g(x) = \max(f_1(x), f_2(x))$ for all $x \in [0,1]$.

A. If f_1 and f_2 are linear, then g is linear

B. If f_1 and f_2 are differentiable, then g is differentiable

C. If f_1 and f_2 are convex, then g is convex

D. None of the above

22. Let $f : \mathbb{R} \to \mathbb{R}$ be the function

$$f(x) = x^3 - 3x \ \forall \ x \in \mathbb{R}.$$

Find the maximum value of $f(x)$ on the set of real numbers x satisfying $x^4 + 36 \leq 13x^2$.

A. 18

B. -2

C. 2

D. 52

23. A monkey is sitting on 0 on the real line in period 0. In every period $t \in \{0, 1, 2, \ldots\}$, it moves 1 to the right with probability p and 1 to the left with probability $1 - p$, where $p \in [\frac{1}{2}, 1]$. Let π_k denote the probability that the monkey will reach positive integer k in some period $t > 0$. The value of π_k for any positive integer k is

A. p^k

B. 1

C. $\frac{p^k}{(1-p)^k}$

D. $\frac{p}{k}$.

24. Refer to the previous question. Suppose $p = \frac{1}{2}$ and π_k now denotes the probability that the monkey will reach any integer k in some period $t > 0$. The value of π_0 is

A. 0

B. $\frac{1}{2^k}$

C. $\frac{1}{2}$

D. 1

25. Suppose $f : \mathbb{R} \to \mathbb{R}$ is a differentiable function with $f'(x) > 0$ for all $x \in \mathbb{R}$ and satisfying the property

$$\lim_{x \to -\infty} f(x) \geq 0.$$

Which of the following must be true?

A. $f(1) < 0$
B. $f(1) > 0$
C. $f(1) = 0$
D. None of the above

26. For what values of x is

$$x^2 - 3x - 2 < 10 - 2x$$

A. $4 < x < 9$
B. $x < 0$
C. $-3 < x < 4$
D. None of the above

27. $\int_e^{e^2} \frac{1}{x(\log x)^3} dx =$
A. $3/8$
B. $5/8$
C. $6/5$
D. $-4/5$

28. The solution of the system of equations

$$\begin{aligned} x - 2y + z &= 7 \\ 2x - y + 4z &= 17 \\ 3x - 2y + 2z &= 14 \end{aligned}$$

is

A. $x = 4, y = -1, z = 3$
B. $x = 2, y = 4, z = 3$
C. $x = 2, y = -1, z = 5$
D. none of the above.

29. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be a twice-differentiable function with non-zero second partial derivatives. Suppose that for every $x \in \mathbb{R}$, there is a unique value of y, say $y^*(x)$, that solves the problem

$$\max_{y \in \mathbb{R}} f(x, y).$$

Then y^* is increasing in x if

A. f is strictly concave

B. f is strictly convex

C. $\frac{\partial^2 f}{\partial x \partial y} > 0$

D. $\frac{\partial^2 f}{\partial x \partial y} < 0$.

30.

$$\int 3^{\sqrt{2x+1}} dx =$$

A.

$$\frac{3^{\sqrt{2x+1}}}{\ln 3} + \frac{\sqrt{2x+1}}{\ln 3} + c$$

B.

$$\frac{3^{\sqrt{2x+1}}\sqrt{2x+1}}{\ln 3} - \frac{3^{\sqrt{2x+1}}}{(\ln 3)^2} + c$$

C.

$$\frac{3^{\sqrt{2x+1}}\sqrt{2x+1}}{(\ln 3)^2} - \frac{3^{\sqrt{2x+1}}}{\ln 3} + c$$

D. none of the above.

1. A researcher has 100 hours of work which have to be allocated between two research assistants, Aditya and Gaurav. If Aditya is allocated x hours of work, his utility is, $-(x-20)^2$. If Gaurav is allocated x hours of work, his utility is, $-(x-30)^2$. The researcher is considering two proposals: (I) Aditya does 60 hours and Gaurav 40 hours (II) Aditya does 90 hours and Gaurav 10 hours. Which of the following statements is correct.

   A. Proposal I is Pareto-efficient but Proposal II is not.

   B. Proposal II is Pareto-efficient but Proposal I is not.

   C. Both proposals are Pareto-efficient.

   D. Neither proposal is Pareto-efficient.

2. The industry demand curve for tea is: $Q = 1800 - 200P$. The industry exhibits constant long run average cost (ATC) at all levels of output at Rs 1.50 per unit of output. Which market form(s) – perfect competition, pure monopoly and first-degree price discrimination – has the highest total market (that is, producer + consumer) surplus?

   A. perfect competition

   B. pure monopoly

   C. first degree price discrimination

   D. perfect competition and first degree price discrimination

3. The following information will be used in the next question also. OIL Inc. is a monopoly in the local oil refinement market. The demand for refined oil is

$$Q = 75 - P$$

   where P is the price in rupees and Q is the quantity, while the marginal cost of production is

$$MC = 0.5Q.$$

   The fixed cost is zero. Pollution is emitted in the refinement of oil which generates a marginal external cost (MEC) equal to 31 Rs/unit. What is the level of Q that maximizes social surplus?

A. 50

B. $29\frac{1}{3}$

C. 17.6

D. 44

4. Refer to the previous question. Suppose the government decides to impose a per unit pollution fee on OIL Inc. At what level should the fee (in Rs/unit) be set to produce the level of output that maximizes social surplus? You may use the fact that the marginal revenue is given by: $MR = 75 - 2Q$.

   A. 1/3

   B. 2

   C. 3/4

   D. 5/3

5. Mr. X has an exogenous income W, and his utility from consumption is given by $U(c)$. With probability p, an accident can occur. If it occurs, the monetary equivalent of the damage is T. Mr. X can however affect the accident probability, p, by taking prevention effort, e. In particular, e can take two values: 0, and $a > 0$. Assume that $p(0) > p(a)$. Let us also assume that the utility cost of effort is Ae^2. Calculate the value of A below which effort will be undertaken.

   A. $\frac{[p(a)-p(0)][u(W-T)-U(W)]}{a^2}$

   B. $\frac{p(a)-p(0)}{u(W-T)-U(W)}$

   C. $\frac{p(a)p(0)a^2}{u(W-T)-u(W)}$

   D. $\frac{p(a)/p(0)}{u(W-T)/u(W)}a^2$

6. Suppose Mr. X maximizes inter-temporal utility for 2 periods. His total utility is given by

$$\log(c_1) + \beta \log(c_2)$$

where $\beta \in (0, 1)$ and c_1 and c_2 are his consumption in period 1 and period 2, respectively. Suppose he earns a wage only in period

1 and it is given by W. He saves for the second period on which he enjoys a gross return of $(1 + r)$ where $r > 0$ is the net interest rate. Suppose the government implements a scheme where $T \geq 0$ is collected from agents (thus also from Mr. X) in the first year, and gives the same amount, T, back in the second period. What is the optimum T for which his total utility is maximized?

A. $T = 0$

B. $T = \frac{W}{2\beta}$

C. $T = \frac{\beta W}{2(1-\beta)}$

D. $T = \frac{W}{2(1-\beta)}$

7. Suppose there is one company in an economy which has a fixed supply of shares in the short run. Suppose there is new information that causes expectations of lower future profits. How does this new stock market equilibrium affect final output and the final price level of the economy if you assume that autonomous consumption spending and household wealth are positively related?

A. real GDP increases; price decreases

B. real GDP decreases; price increases

C. real GDP decreases; price decreases

D. real GDP increases; price stays constant.

8. A monopolist faces a demand function, $p = 10 - q$. It has two plants at its disposal. The cost of producing q_1 in the first plant is $300 + q_1^2$, if $q_1 > 0$, and 0 otherwise. The cost of producing q_2 in the second plant is $200 + q_2^2$, if $q_2 > 0$, and 0 otherwise. What are the optimal production levels in the two plants?

A. 10 units in both plants,

B. 20 units in the first plant and 10 units in the second plant

C. 0 units in the first plant and 15 units in the second plant

D. None of the above.

9. Consider a firm facing three consumers, 1, 2 and 3, with the following valuations for two goods, X and Y (All consumers consume at most 1 unit of X and 1 unit of Y.)

Consumers	X	Y
1	7	1
2	4	5
3	1	6

The firm can produce both goods at a cost of zero. Suppose the firm can supply both goods at a constant per unit price of p_X for X, and p_y for Y. It can also supply the two goods as a bundle, for a price of p_{XY}. The optimal vector of prices (p_X, p_y, p_{XY}) is given by

A. (7,6,9).

B. (4,1,4).

C. (7,7,7).

D. None of the above.

10. Two individuals, Bishal (B) and Julie (J), discover a stream of mountain spring water. They each separately decide to bottle some of this water and sell it. For simplicity, presume that the cost of production is zero. The market demand for bottled water is given by $P = 90 - 0.25Q$, where P is price per bottle and Q is the number of bottles. What would Bishal's output Q_B, Julie's output Q_J, and the market price be if the two individuals behaved as Cournot duopolists?

A. $Q_B = 120$; $Q_J = 120$; $P = 42$

B. $Q_B = 90$; $Q_J = 90$; $P = 30$

C. $Q_B = 120$; $Q_J = 120$; $P = 30$

D. $Q_B = 100$; $Q_J = 120$; $P = 30$

11. The next three questions (**11, 12, 13**) are to be answered together. Consider the following model of a closed economy

$$\begin{aligned}
\triangle Y &= \triangle C + \triangle I + \triangle G \\
\triangle C &= c\triangle Y_d \\
\triangle Y_d &= \triangle Y - \triangle T \\
\triangle T &= t\triangle Y + \triangle T_0
\end{aligned}$$

where $\triangle Y$ = change in GDP, $\triangle C$ = change in consumption, $\triangle I$ = change in private investment, $\triangle G$ = change in government spending, $\triangle Y_d$ = change in disposable income (i.e., after tax income), $\triangle T$ = the change in total tax collections, t is the tax rate between $(0, 1)$, and $\triangle T_0$ = the change in that portion of tax collections that can be altered by government fiscal policy measures. The value of the balanced budget multiplier (in terms of G and T_0) is given by:

A. $\frac{1}{1-c(1-t)}$

B. $\frac{-c}{1-c(1-t)}$

C. $\frac{1-c}{1-c(1-t)}$

D. none of the above.

12. Refer to the previous question. Suppose the marginal propensity to consume, $c = .8$, and $t = .375$. The value of the government expenditure multiplier is

A. 2,

B. -1.6

C. .4

D. .5

13. Refer to the previous two questions. Suppose the marginal propensity to consume, $c = .8$, and $t = .375$. The value of the tax multiplier (with respect to T_0) is

A. -1.6

B. 2

C. .4

D. .3

14. In the IS-LM model, a policy plan to increase national savings (public and private) without changing the level of GDP, using any combination of fiscal and monetary policy involves

    A. contractionary fiscal policy, contractionary monetary policy

    B. expansionary fiscal policy, contractionary monetary policy

    C. contractionary fiscal policy, expansionary monetary policy

    D. expansionary fiscal policy, expansionary monetary policy

15. Consider the IS-LM-BP model with flexible exchange rates but with no capital mobility. Consider an increase in the money supply. At the new equilibrium, the interest rate is ______, the exchange rate is______, and the level of GDP is______, respectively.

    A. higher, lower, higher

    B. lower, higher, higher

    C. lower, higher, lower

    D. higher, lower, lower

16. Consider a Solow model of an economy that is characterized by the following parameters: population growth, n; the depreciation rate, δ; the level of technology, A; and the share of capital in output, α. Per-capita consumption is given by $c = (1 - s)y$ where s is the exogenous savings rate, and $y = Ak^{\alpha}$, where y denotes output per-capita, and k denotes the per-capita capital stock. The economy's golden-rule capital stock is determined by which of the following conditions?

    A. $\frac{\partial c}{\partial k} = Ak^{\alpha} - (n + \delta)k = 0$

    B. $\frac{\partial c}{\partial k} = \alpha Ak^{\alpha-1} - (n + \delta) = 0$

    C. $\frac{\partial c}{\partial k} = (n + \delta)k - sAk^{\alpha} = 0$

    D. none of the above.

17. In the Ramsey model, also known as the optimal growth model, with population growth, n, and an exogenous rate of growth of technological progress, g, the steady-state growth rates of aggregate output, Y, aggregate capital, K, and aggregate consumption, C, are

    A. $0, 0, 0$

    B. $n+g, n+g, n+g$

    C. $g, n+g, n$

    D. $n+g, n+g, g$

18. Consider the standard formulation of the Phillips Curve,

$$\pi_t - \pi_t^e = -\alpha(u_t - u_n)$$

where π_t is the current inflation rate, π_t^e is the expected inflation rate, α is a parameter, and u_n is the natural rate of unemployment. Suppose the economy has two types of labour contracts: a proportion, λ, that are indexed to actual inflation, π_t, and a proportion, $1-\lambda$, that are not indexed and simply respond to last year's inflation, π_{t-1}. Wage indexation (relative to no indexation) will ________ the effect of unemployment on inflation.

    A. strongly decrease

    B. increase

    C. not change

    D. mildly decrease

19. Consider a Harrod-Domar style growth model with a (i) Leontieff aggregate production function, (ii) no technological progress, and (iii) a constant savings rate. Let K and L denote the level of capital and labor employed in the economy. Output, Y, is produced according to

$$Y = \min\{AK, BL\}$$

where A and B are positive constants. Let $\bar{L}$ be the full employment level. Under what condition will there be positive unemployment?

    A. $AK > B\bar{L}$

    B. $AK < B\bar{L}$

C. $AK = B\bar{L}$

D. none of the above.

20. The next two questions (**20** and **21**) are to be answered together. People in a certain city get utility from driving their cars but each car releases k units of pollution per km driven. The net utility of each person is his or her utility from driving, v, minus the total pollution generated by everyone else. Person i's net utility is given by

$$U_i(x_1, ..., x_n) = v(x_i) - \sum_{\substack{j=1 \\ j \neq i}}^{n} kx_j$$

where x_j is km driven by person j, n is the city population, and the utility of driving v has an inverted U-shape with $v(0) = 0$, $lim_{x \to 0^+} v'(x) = \infty$, $v''(x) < 0$, and $v(\bar{x}) = 0$ for some $\bar{x} > 0$. In an unregulated city, an increase in population will

A. increase the km driven per person

B. decrease the km driven per person

C. leave the km driven per person unchanged

D. may or may not increase the km driven per person.

21. Refer to the information given in the previous question. A city planner decides to impose a tax per km driven and sets the tax rate in order to maximize the total net utility of the residents. Then, if the population increases, the optimal tax will

A. increase

B. decrease

C. stay unchanged

D. may or may not increase.

22. The production function

$$F(L, K) = (L + 10)^{1/2} K^{1/2}$$

has

A. increasing returns to scale
B. constant returns to scale
C. decreasing returns to scale
D. none of the above.

23. Consider the production functions

$$F(L, K) = L^{1/2}K^{2/3} \text{ and } G(L, K) = LK.$$

where L denotes labour and K denotes capital.

A. F is consistent with the law of diminishing returns to capital but G is not.
B. G is consistent with the law of diminishing returns to capital but F is not.
C. Both F and G are consistent with the law of diminishing returns to capital
D. Neither F nor G is consistent with the law of diminishing returns to capital

24. A public good is one that is non-rivalrous and non-excludable. Consider a cable TV channel and a congested city street.

A. A cable TV channel is a public good but a congested city street is not
B. A congested city street is a public good but a cable TV channel is not
C. Neither is a public good
D. Both are public goods.

25. Firm A's cost of producing output level $y > 0$ is, $c_A(y) = 1 + y$ while Firm B's cost of producing output level y is, $c_B(y) = y(1-y)^2$

A. A can operate in a perfectly competitive industry but B cannot
B. B can operate in a perfectly competitive industry but A cannot

C. Neither could operate in a perfectly competitive industry

D. Either could operate in a perfectly competitive industry.

26. Suppose we generically refer to a New Keynesian model as a model with a non vertical aggregate supply (AS) curve. Under sticky prices, the AS curve will be ______, and under sticky wages, the AS curve will be ______, respectively.

A. horizontal, upward sloping

B. upward sloping, upward sloping

C. downward sloping, horizontal

D. upward sloping, horizontal

27. With perfect capital mobility, and ______, monetary policy is ______ at influencing output.

A. fixed exchange rates, effective

B. fixed exchange rates, ineffective

C. flexible exchange rates, ineffective

D. none of the above are correct

28. The next three questions (**28, 29** and **30**) use the following information. Consider an economy with two goods, x and y, and two consumers, A and B, with endowments (x, y) given by $(1, 0)$ and $(0, 1)$ respectively. A's utility is

$$U_A(x, y) = x + 2y$$

while B's utility is

$$U_B(x, y) = 2x + y.$$

Using an Edgeworth box with x measured on the horizontal axis and y measured on the vertical axis, with A's origin in the bottom-left corner and B's origin in the top-right corner, the set of Pareto-optimal allocations is

A. a straight line segment

B. the bottom and right edges of the box

C. the left and top edges of the box

D. none of the above.

29. Referring to the information given in the previous question, the following allocations are the ones that may be achieved in some competitive equilibrium.

A. (0,1)

B. The line segment joining $(0, 1/2)$ to $(0, 1)$ and the line segment joining $(0, 1)$ to $(1/2, 1)$

C. The line segment joining $(1/2, 0)$ to $(1, 0)$ and the line segment joining $(1, 0)$ to $1, 1/2)$

D. (1,0)

30. Referring to the information given in the previous two questions, if the price of y is 1, then the price of x in a competitive equilibrium

A. must be 1/2

B. must be 1

C. must be 2

D. could be any of the above.