Solutions: DSE entrance 2017

October 9, 2018

DSE Super 20

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Problem 1

Person A lexicographically prefers x to y. A's indifference curves in the (x, y) space are

(A) vertical lines

(B) horizontal lines

(C) diagonal lines

(D) none of the above

d.
The indifference curves for a lexicographic preference in 2 goods are singleton sets. This is because for and two points in the consumption set $Z_1 = (x_1, y_1)$ and $Z_2(x_2, y_2)$ such that Z_1 and Z_2 are not the same, either $Z_1/succZ_2$ or $Z_2 \succ Z_1$. Thus the indifference curve through Z_1 cannot pass through any other point in the consumption set and hence is a singleton set, that is to say that each point is the indifference curve of itself.

Problem 2

Consider a person A as described above. Consider a bundle (2,1) and a sequence of bundles (s^n) such that each bundle is strictly preferred to (2,1). If this sequence of bundles converges to a bundle s, then

(A) $s \succ (2, 1)$

(B) $s \sim (2, 1)$

(C) $s \prec (2, 1)$

(D) any of the above

D.
Let the sequence be defined as $s^n = \left(2, \dfrac{3}{1+\frac{1}{n}}\right)$ for all $n \geq 1$. Then each element of the sequence is strictly preferred to (2,1). The limit of this sequence is $s = (2, 3)$ which is such that $s \succ (2, 1)$.
Let the sequence be defined as $s^n = \left(2, \dfrac{2}{2-\frac{1}{n}}\right)$ for all $n \geq 1$. Then each element of the sequence is strictly preferred to (2,1). The limit of this sequence is $s = (2, 1)$ which is such that $s \sim (2, 1)$.
Let the sequence be defined as $s^n = \left(\dfrac{2}{2-\frac{1}{n}}, 0\right)$ for all $n \geq 1$. Then each element of the sequence is strictly preferred to (2,1). The limit of this sequence is $s = (2, 0)$ which is such that $s \prec (2, 1)$.
Thus all of the first 3 options can be correct.

Problem 3

Consider an economy with 2 agents, A and B, and two goods, x and y. Both agents treat the two goods as perfect compliments. Suppose the total endowment of this economy is (4,2). Which of the following allocation is not Pareto Optimal?

(A) A gets (1,1) and B gets (1,1)

(B) A gets (2,1) and B gets (1.5,1)

Problem 3 continued on next page...

(C) A gets (0.5,1.5) and B gets (3,0.5)

(D) A gets (3,2) and B gets (0,0)

C.
An allocation where A gets (1,1.5) and B gets (2.5,0.5) is a pareto improvement of allocation in option C. Thus C is not a pareto optimal allocation.

The next four questions are based on the following information:
Consider an economy with agents A and B and goods x and y, with the respective allocations as (0,1) and (2,0). The agents can consume only non-negative amounts of x and y.

Problem 4

Suppose A lexicographically prefers x and B considers x and y to be perfect substitutes. The competitive equilibrium allocation for this economy is

(A) A gets (0,1) and B gets (2,0)

(B) A gets (2,0) and B gets (0,1)

(C) A gets (1.5,0) and B gets (0.5,1)

(D) A gets (1,0) and B gets (1,1)

D.
Given any price vector $(p, 1)$, A solves $p.x_A = 0.p + 1.1 \implies x_A = \frac{1}{p}$. For B, we will have 3 cases.
If $p > 1$, then $x_B = 0$ and in this case $x_A < 1$. Hence the market for good x won't clear.
If $p < 1$, then $x_B = \frac{2}{p}$ and $x_A > 1$. Thus total demand for good x is more than 1 and the market won't clear.
If $p = 1$, then $x_B + y_B = 2$ and $x_A = 1$. Then we have $x_B = 0$ and $y_B = 2$. Then market for both goods will clear. Hence this is the equilibrium allocation.

Problem 5

Suppose A lexicographically prefers x and B considers x and y to be perfect substitutes. The set of all competitive equilibrium prices consists of all $p_x > 0$ and $p_y > 0$ such that

(A) $p_x/p_y = 1$

(B) $p_x/p_y \geq 1$

(C) $p_x/p_y \leq 1$

(D) $p_x/p_y > 1$

A.
Solved in the previous question.

Problem 6

Suppose A lexicographically prefers y and B considers x and y to be perfect substitutes. The set of all competitive equilibrium prices consists of all $p_x > 0$ and $p_y > 0$ such that

(A) $p_x/p_y = 1$

(B) $p_x/p_y \geq 1$

(C) $p_x/p_y \leq 1$

(D) $p_x/p_y > 1$

C.
let the price vector be $(p, 1)$. Then agent B, who has all of good y, is willing to trade y for x only when $p \leq 1$. At any such price vector agent A will trade all of his x to get p amount of y. Thus the equilibrium allocation for any prive vector $(p, 1)$ such that $p \leq 1$ will be $(0, p)$ and $(1, 2 - p)$.

Problem 7

Suppose A lexicographically prefers y and B considers x and y to be perfect complements. The set of competitve equilibrium allocations

(A) $p_x/p_y = 1$

(B) $p_x/p_y \geq 1$

(C) $p_x/p_y \leq 1$

(D) $p_x/p_y > 1$

A.
Solved in the previous question.

The next 2 questions are based on the following information:
Suppose A is selling the Taj Mahal by the following auction procedure. There are two bidders, 1 and 2. Each bidder has a valuation v_i of the Taj and submits a bid b_i in a sealed envelope. The Taj is given to the bidder who submits the highest bid; if both bidders submit the same bid, then each gets the Taj with equal probability. If bidder i wins, then she pays the price $\min\{b_1, b_2\}$ and gets payoff $v_i - \min\{b_1, b_2\}$. If bidder i loses, then she pays nothing and her payoff is 0. Bidder i's valuation v_i is known only to bidder i and her bid b_i may or may not match v_i.

Problem 8

In order to maximize her payoff bidder i must bid

(A) $b_i = v_i$

(B) $b_i < v_i$

(C) $b_i \leq v_i$

(D) $b_i \geq v_i$

a
In the second price auction game, each bidder must bid exactly the valuation. This is because the payoff from winning is independent of the amount of bid, because if bidder 1 wins, her bid must have been higher. THus $\min\{b_1, b_2\} = b_2$ and the payoff is $v_1 - b_2$, which is independent of b_1. So expected payoff is maximized by bidding a the highest possible value, which must be equal to v_1. She will not bid a value higher than her valuation because then there is a probability of she winning and paying a price higher than her valuation. This will happen if $b_1 > b_2 > v_1$. Hence both bidders will bid their individual valuations

Problem 9

If b_i is the optimal bid for bidder i, then

(A) it varies with bidder i's belief about the other bidder's valuation

(B) it varies with bidder i's belief about the other bidder's bid

(C) Both (A) and (B)

(D) Neither (A) nor (B)

d
Her bid is independent of the valuation or the bid of the other agent, as seen in the solution above.

Problem 10

The Nash equilibrium of the Cournot duopoly model is a pair of quantities (x_1, x_2) such that

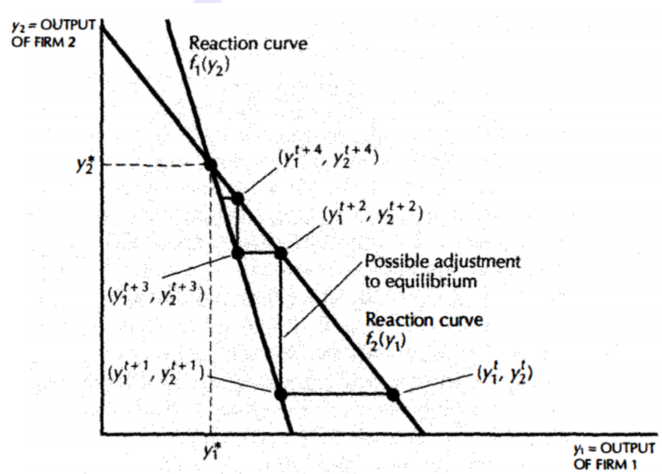
(A) the best response curves of firms 1 and 2 are tangential at (x_1, x_2)

(B) isoprofit curves of firms 1 and 2 are tangential at (x_1, x_2)

(C) an isoprofit curve of each firm is tangential to the best response curve of the other firm at (x_1, x_2)

(D) none of the above

d
The Cournot equilibrium is the intersection of the best response curves, as shown in the figure below



Problem 11

Consider the Stackelberg duopoly model with firm 1 choosing quantity x_1 first. Firm 2 observes x_1 and sets quantity x_2 thereafter. The equilibrium outcome of this game is a pair of quantities (x_1, x_2) such that

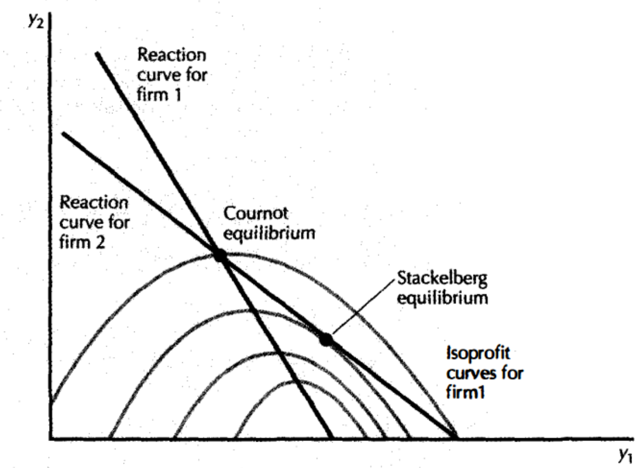
(A) isoprofit curves of firms 1 and 2 are tangential at (x_1, x_2)

(B) isoprofit curve of firm 1 is tangential to the best response curve of firm 2 at (x_1, x_2)

(C) isoprofit curve of firm 1 is tangential to the best response curve of firm 2 at (x_1, x_2)

(D) none of the above

b



Stackelberg equilibrium. Firm 1, the leader, chooses the point on firm 2's reaction curve that touches firm 1's lowest possible isoprofit line, thus yielding the highest possible profits for firm 1.

Problem 12

Consider the following game:

	L	R
T	(x,x)	(b,x)
B	(y,b)	(a,a)

Which of the following statements is **TRUE** when $y > x > a > b$?

(A) (B, R) is the unique Nash Equilibrium.

(B) (B, R) is one of the many Nash Equilibrium

(C) Both (B, R) and (T, L) are Nash Equilibria

(D) none of the above

a

Playing B is the dominant strategy for the row player and playing R is the dominant strategy for the column player. Thus (B, R) is the unique NE.

Problem 13

Consider a consumer with utility function $u(x, y, z) = y \min\{x, z\}$. The prices of all three goods are the same. The consumer has Rs. 100 to spend on these 3 goods. The demands will be such that

(A) $y < x = z$

(B) $y > x = z$

(C) $x = y = z$

(D) none of the above

b

Let price of each good be p. Imagine a composite good $k = \min\{x, z\}$, that is x and z combine in equal proportions to yield the same amount of k. Then $p_k = p_x + p_z = 2p$. Now the utility function becomes $u(y, k) = yk$ where $p_y = p$ and $p_k = 2p$. Thus the consumer will consume more of y and less of k. So we have $x > k \implies y > x = z$

Problem 14

A firm uses two inputs to produce its output. For all positive input prices, the firm employs an input combination of the form $(x, \alpha x)$ where $\alpha > 0$ is a constant.
Which of the following production functions could represent this firm's technology?

(A) $f(x, y) = \min\{x^\alpha, y\}$

(B) $f(x, y) = \min\{\alpha x, y\}$

(C) $f(x, y) = \min\{x, \alpha y\}$

(D) $f(x, y) = \min\{x, y^\alpha\}$

b
$f(x, y) = \min\{\alpha x, y\}$ satisfies the condition $\alpha x = y$ for the given input point $(x, \alpha x)$.

Problem 15

Suppose there are 2 telecom firms 1 and 2 who have paid fees $k_1 > 0$ and $k_2 > 0$ for telecom spectrum. Let $k_1 > k_2$. They produce an identical good (telecom service) at an identical average cost of production $c > 0$. If they engage in Bertrand competition, then the Nash equilibrium prices (p_1, p_2) are such that

(A) $p_1 > p_2 > c$

(B) $p_1 > p_2 = c$

Problem 15 continued on next page. . .

(C) $p_1 = p_2 = c$

(D) are indeterminate

c
The Bertrand equilibrium is independent of the fixed costs and as long as marginal costs are same, the equilibrium must be $p_1 = p_2 = c$.

Problem 16

Consider the following game with players 1 and 2; payoffs are denoted by (a, b) where a is 1's payoff and b is 2's payoff. First, player 1 chooses either U or D. If she plays D, then the game ends and the payoff are (1, 0). If she plays U, then player 2 chooses either U or D. If he plays D, then the game ends and the payoffs are (0, 2). If he plays U, then player 1 again chooses either U or D. The game ends in both cases. If player 1 chooses D, then the payoffs are (4, 0). If player 1 chooses U, then the payoffs are (3, 3).

(A) This game has a unique Nash equilibrium

(B) This game has a unique subgame perfect equilibrium

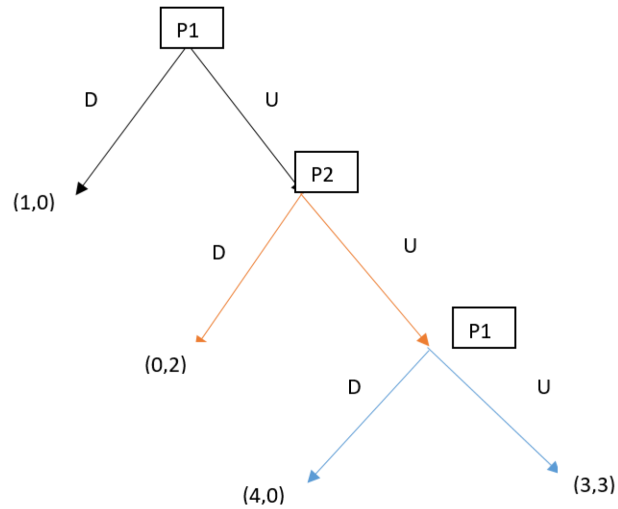
(C) This game has no subgame perfect equilibrium

(D) This game has multiple subgame perfect equilibria.

Problem 16 continued on next page...

b

The extensive form of the game is as follows:



As can be seen from backward induction, the game ends in the first stage itself with P1 playing D. The matrix form of the game is represented below. The * marked entries are the equilibrium in this form.

	U	D
(U,U)	(3,3)	(0,2)
(U,D)	(4,0)	(0,2)
(D,D)	(1,0)	(1,0)*
(D,U)	(1,0)	(1,0)*

Of these, the strategy profile $((D, D), D)$ is the sole Subgame perfect NE. THis is because of somehow the game reaches the final node, this profile ensures that P1 choose the option D instead of U, which is the rational choice.

Problem 17

The interval $(0, 1)$ can be represented as

(A) the union of countable collection of closed intervals

(B) the intersection of countable collection of closed intervals

(C) both (A) and (B)

(D) neither (A) and (B)

A

An arbitrary intersection of closed sets is closed, and a finite union of closed sets is closed. Consider an infinite union of closed intervals of the form $I_n = \left[\frac{1}{n}, 1 - \frac{1}{n}\right]$

Problem 18

The interval $[0,1]$ can be represented as

(A) the union of countable collection of open intervals

(B) the intersection of countable collection of open intervals

(C) both (A) and (B)

(D) neither (A) and (B)

B
An arbitrary union of open sets is open, and a finite intersection of open sets is open. Consider an infinite intersection of open intervals of the form $I_n = \left(-\frac{1}{n}, 1+\frac{1}{n}\right)$

Problem 19

A function $f : \mathbb{R}^n \to \mathbb{R}$ is a convex function if and only if

(A) $\{(x,y) \in \mathbb{R}^n \times \mathbb{R} | y \geq f(x)\}$ is a convex set

(B) $\{(x,y) \in \mathbb{R}^n \times \mathbb{R} | y \leq f(x)\}$ is a convex set

(C) $\{(x,y) \in \mathbb{R}^n \times \mathbb{R} | y \geq f(x)\}$ is a concave set

(D) $\{(x,y) \in \mathbb{R}^n \times \mathbb{R} | y \leq f(x)\}$ is a concave set

A
A function is convex if its epigraph (the set of points on or above the graph of the function) is a convex set

Problem 20

A function $f : \mathbb{R}^n \to \mathbb{R}$ is a quasi-convex function if and only if

(A) $\{(x,y) \in \mathbb{R}^n \times \mathbb{R} | y \geq f(x)\}$ is a convex set

(B) $\{x \in \mathbb{R}^n | y \geq f(x)\}$ is a convex set

(C) $\{(x,y) \in \mathbb{R}^n \times \mathbb{R} | y \geq f(x)\}$ is a concave set

(D) $\{x \in \mathbb{R}^n | y \leq f(x)\}$ is a convex set

D
A (strictly) quasiconvex function has (strictly) convex lower contour sets, while a (strictly) quasiconcave function has (strictly) convex upper contour sets.

Problem 21

If

$$A = \{(x, y) \in \mathbb{R}^2 | x \geq 0, y \geq 0, xy \geq 1\}$$
$$B = \{(x, y) \in \mathbb{R}^2 | x \leq 0, y \geq 0, xy \leq -1\}$$

and

$$C = \{a + b | a \in A, b \in B\}$$

(A) $\{(x, y) \in \mathbb{R}^2 | x = 0, y \geq 0\}$ is a subset of C

(B) $\{(x, y) \in \mathbb{R}^2 | x = 0, y > 0\}$ is a subset of C

(C) $\{(x, y) \in \mathbb{R}^2 | x \geq 0, y = 0\}$ is a subset of C

(D) $\{(x, y) \in \mathbb{R}^2 | x > 0, y = 0\}$ is a subset of C

B

Set A is the set of points in the first quadrant on or above the $xy = 1$ curve. Set is the set of points on or above the $xy = -1$ curve in the second quadrant. In fact, set B looks like a mirror image of set A about the Y-axis.

Now set C is formed by adding a vector from A with a vector from B. Since the y coordinate of any vector from A or B has to be strictly positive, addition of the y coordinates has to be strictly positive as well. Thus the y in set C can never be zero.

Problem 22

$\lim_{n\to\infty} \left(1 + \frac{1}{2n}\right)^{5n}$ equals

(A) e^2

(B) $e^{2.5}$

(C) ∞

(D) $e^{1.5}$

B

Let the limit be equal to L. Then taking logs we get

$$\ln L = \lim_{n\to\infty} 5n \ln\left(1 + \frac{1}{2n}\right) = \lim_{n\to\infty} \frac{\ln\left(1 + \frac{1}{2n}\right)}{\frac{1}{5n}}$$

Now the last expression is in $\frac{0}{0}$ form. SO applying **L,Hopital** rule

$$\ln L = \lim_{n\to\infty} \frac{1}{\left(1 + \frac{1}{2n}\right)} \left(-\frac{1}{2n^2}\right)(-5n^2) = \frac{5}{2}$$

Problem 23

$$f(x) = \begin{cases} \ln x & \text{if } 0 < x < 1 \\ ax^2 + b & \text{if } 1 \leq x < \infty \end{cases}$$

such that $f(2) = 3$. The function is continuous if

(A) $a = 2$, $b = -1$

(B) $a = -1$, $b = 2$

(C) $a = -1$, $b = 1$

(D) $a = 1$, $b = -1$

D

$$f(1^-) = 0 \implies f(1^+) = 0 \implies a + b = 0$$
$$f(2) = 3 \implies 4a + b = 3$$

Solving the above two equations gives $a = 1$, $b = -1$

Problem 24

A and B are $n \times n$ matrices such that $A + B = AB$, then

(A) $AB = BA$

(B) $AB \neq BA$

(C) $B = A^{-1}$

(D) $B = A^T$

A

$$A + B = AB \implies (A - 1)(B - 1) = AB - A - B + 1 = 1$$

Thus we get that $(A - 1)$ and $(B - 1)$ are inverse of each other. Thus we get

$$(B - 1)(A - 1) = BA - A - B + 1$$

must also equal 1. So $BA - A - B = 0 \implies BA = A + B = AB$.

Problem 25

If B is an $n \times n$ real matrix, then

(A) $B^T B$ is negative definite

(B) $B^T B$ is positive definite

(C) $B^T B$ is negative semidefinite

(D) $B^T B$ is positive semidefinite

D
An $n \times n$ matrix A is positive semi definite if for any $1 \times n$ vector v we have $v^T AV \geq 0$. For any real valued $n \times n$ matrix B, $v^T B^T Bv = (Bv)^T Bv = Bv.Bv \geq 0$, where $Bv.Bv$ is the dot product of the vector obtained by the matrix multiplication Bv.

Problem 26

Consider a twice differentiable function $f : \mathbb{R} \to \mathbb{R}$ and $a, b \in \mathbb{R}$ such that $a < b$ and $f(a) = 0 = f(b)$ and $D^2 f(x) + Df(x) - 1 = 0$ for every $x \in [a, b]$. Then,

(A) $f(x) \leq 0$ for every $x \in [a, b]$

(B) $f(x) \geq 0$ for every $x \in [a, b]$

(C) $f(x) = 0$ for every $x \in [a, b]$

(D) f must take positive and negative values on the interval $[a, b]$

A
$f(a) = f(b) = 0$. So by Rolle theorem, there exists a $y \in (a, b)$ such that $f'(y) = 0$. Now the equation $D^2 f(x) + Df(x) - 1 = 0$ is same as $f''(x) + f'(x) = 1$. So at point y we have $f''(y) = 1 > 0$. Hence the function will be convex at all the stationary points. Thus we have $f(x) \geq 0$ at all points.

Problem 27

Suppose $f : \mathbb{R}_+ \to \mathbb{R}$ is decreasing and differentiable. If $F : \mathbb{R}_+ \to \mathbb{R}$ satisfies $F(x) = \int_0^x f(t)dt$, then F is

(A) convex

(B) concave

(C) increasing

(D) decreasing

D
$F'(x) = f(x) \implies F''(x) = f'(x)$. Now since f is decreasing, $f'(x) < 0 \implies F''(x) < 0$. SO the function is concave

Problem 28

If real numbers p and q satisfy $0 < q < p$, then the following is true for the numbers p, q, $p + q$ and $q - p$

(A) their mean equals their median

(B) their mean is greater than the median

(C) their mean is less than their median

(D) there is insufficient information to compare their mean and their median

C
Given than $0 < q < p$ we get $(q - p) < p < q < (p + q)$. So the median is $\frac{p+q}{2}$. However the mean is $\frac{(q-p) + (p) + (q) + (p+q)}{4} = \frac{3q+p}{4}$. Since $q < p$ we have $\frac{p+q}{2} > \frac{3q+p}{4}$.

Problem 29

If a binomial random variable X has expectation 7 and variance 2.1, then the probability that $X = 11$ is

(A) $462(0.7)^5(0.3)^6$

(B) 0

(C) $11(0.7)^{11}$

(D) $462(0.7)^6(0.3)^5$

B

We have $np = 7$ and $np(1-p) = 2.1$. This gives $p = 0.7$ and $n = 10$. Since $n = 10$, the maximum value X can take is 10. Therefore $P(X = 11) = 0$.

Problem 30

A machine starts operating at time 0 and fails at a random time T. The distribution of T has density $f(t) = \frac{1}{3}e^{-t/3}$ for $t > 0$. The machine will not be monitored until time $t = 2$. The expected discovery time of the machine's failure is

(A) $2 + e^{-6}/3$

(B) $2 - 2e^{-2/3} + 5e^{-4/3}$

(C) $2 + 3e^{-2/3}$

(D) 3

C

$$\begin{aligned} E(T) &= 2.\int_0^2 \frac{1}{3}e^{-t/3}dt + \int_2^\infty t\frac{1}{3}e^{-t/3}dt \\ &= 2[1 - e^{-2/3}] + [2e^{2/3} + 3e^{-2/3}] \\ &= 2 + 3e^{-2/3} \end{aligned}$$

Problem 31

An insuree has an insurance policy against a random loss $X \in [0,1]$. If loss X occurs then the insurer pays $X - C$ to the insuree, who bears the remaining loss $C \in (0,1)$. The loss X is a continuous random variable with density function

$$f(x) = \begin{cases} 2x, & 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$$

If the probability of insurance payment being less than $1/2$ is 0.64, then C is

(A) 0.1

(B) 0.3

(C) 0.4

(D) 0.6

C

$$P(X \leq t) = 0.64 \implies \int_0^t 2x dx = 0.64 \implies t^2 = 0.64 \implies t = 0.8$$

Now $P(X \leq 0.8) = P(X - C \leq 0.5 \implies C = 0.3$.

Problem 32

A tour operator has a bus that can accommodate 20 tourists. The operator knows that tourists may not show up, so he sells 21 tickets. The probability that an individual tourist will not show up is 0.02, independent of all other tourists. Each ticket costs 50, and is non-refundable if a tourist fails to show up. If a tourist shows up and a seat is not available, the tour operator has to pay 100 to that tourist. The expected revenue of the tour operator is

(A) 950

(B) 967

(C) 976

(D) 985

D

$$E(R) = 21(50) - (0.98)^{21}(100) = 1050 - (1 - 0.02)^{21}(100) \approx 1050 - 100(1 - 0.42) = 1050 - 58 = 992$$

So the closest option is 985.

Problem 33

An insurance policy holder can submit upto 5 claims. The probability that the policyholder submits exactly n claims is p_n for $n \in \{0, 1, 2, 3, 4, 5\}$. It is known that

1) The difference between p_n and p_{n+1} is constant for all $n \neq 5$.

2) 40% of the policyholders submit 0 or 1 claim.

What is the probability that the policyholder submits 4 or 5 claims?

(A) 0.06

(B) 0.19

(C) 0.26

(D) 0.34

C

Let the values of p_n be $\{a-5d, a-3d, a-d, a+d, a+3d, a+5d\}$ for $n = \{0, 1, 2, 3, 4, 5\}$ respectively. Then since the sum of probabilities must equal 1, we have $6a = 1 \implies a = \frac{1}{6}$. Also $P(n = 0, 1) = 2a - 8d = 0.4 \implies -8d = 0.4 - \frac{1}{3} \implies 8d = \frac{-1}{15}$. Thus $P(n = 4, 5) = 2a + 8d = \frac{1}{3} - \frac{1}{15} = \frac{4}{15} = 0.26$.

Problem 34

Suppose n students are asked to solve a problem at time $t = 0$. The random time to completion for student i is $T_i \geq 0$. Each T_i is uniformly distributed on [0,1]. If $Y = \max\{T_1, \cdots T_n\}$, then the mean of Y is

(A) $[n/(n+1)]^2$

(B) $n/2(n+1)$

(C) $n/n+1$

(D) $2n/n+1$

C

The probability that Y takes a value y is $P(Y = y) = n.y^{n-1}$. This is because for the max to take a value y, one of the n variables has to take a value y and all the others must be less than or equal to y. Therefore the expected value is

$$E(Y) = \int_0^1 y(n.y^{(n-1)})dy = \frac{n}{n+1}[y^{n+1}]_0^1 = \frac{n}{n+1}$$

Problem 35

A hospital determines that N, the number of patients in a week, is a random variable with $P[N = n] = 2^{-n-1}$, where $n \geq 0$. The hospital also determines that the number of patients in week is independent of the number of patients in any other week. The probability that there are exactly 7 patients during a two week period is

(A) 1/32

(B) 1/64

(C) 1/128

(D) 1/256

B

Let the number of patients in the two consecutive weeks be n and m respectively. Then the probability that

$$\begin{aligned} P(n+m=7) &= P(n=0,m=7) + P(n=1,m=6) \cdots P(n=6,m=1) + P(n=7,m=0) \\ &= \frac{1}{2}\frac{1}{2^8} + \frac{1}{2^2}\frac{1}{2^7} \cdots \frac{1}{2^7}\frac{1}{2^2} + \frac{1}{2^8}\frac{1}{2} \\ &= \frac{1}{2^9} + \frac{1}{2^9} + \cdots \frac{1}{2^9} + \frac{1}{2^9} \\ &= 8.\frac{1}{2^9} = \frac{1}{64} \end{aligned}$$

The next two questions are based on the following information.

X and Y are random variables and their joint PDF is as follows:

	$Y=1$	$Y=2$	$Y=3$	$Y=4$	$Y=5$	$Y=6$
$X=-1$	0.1	α	0.3	0	0	0
$X=1$	0	0	β	0.1	0.1	0.1

It is known that the expectations of the two random variables are $E(X) = -0.2$ and $E(Y) = 3.2$. Then

Problem 36

The value of α is

(A) 0

(B) 0.1

(C) 0.2

(D) 0.3

C
$P(X=-1)=0.4+\alpha$ and $P(X=1)=0.3+\beta$. This gives $E(X)=-1(0.4+\alpha)+1(0.3+\beta)=\beta-\alpha-0.1=-0.2 \implies \alpha-\beta=0.1$.
Similar calculations for Y gives $E(Y)=0.1+2\alpha+(0.9+3\beta)+0.4+0.5+0.6=2.5+2\alpha+3\beta=3.2$.
Solving the above system of equation simultaneously gives $\alpha=0.2$ and $\beta=0.1$.

Problem 37

The value of β is

(A) 0

(B) 0.1

(C) 0.2

(D) 0.3

B
Solved prviously

Problem 38

There are 3 red and 5 black balls in an urn. You draw two balls in succession without replacing the first ball. The probability that the second ball is red equals

(A) 2/7

(B) 3/8

(C) 5/7

(D) 1/4

B
Suppose you arrange the 8 balls in a random manner, and then pick the balls in the first two position. Then the probability of a position being occupied by a red ball is 3/8. Hence even the second position could have a red ball with 3/8 probability.

Problem 39

Suppose X and Y are random variabes with standard normal distributions. The probability of $X > 1$ is p. The probability of the event $X^2 > 1$ and $Y^3 < 1$ is

(A) $2p(1-p)$

(B) $4p$

(C) $p(1-p)$

(D) $2p^2$

A

$$P(X^2 > 1).P(Y^3 < 1) = P(X > 1 \vee X < -1)[1 - P(Y > 1)] = 2p(1-p)$$

Problem 40

Suppose 1/10 of the population has a disease. If a person has the disease, then a test detects it with probability 8/10. If a person does not have the disease, then the test incorrectly shows the presence of the disease with probability 2/10. What is the probability that the person tested has the disease if the test indicates the presence of the disease?

(A) 1

(B) 9/13

(C) 4/13

(D) 7/13

C
Let A be the event that a person has the disease and B be the event that the test is positive. Then

$$P(A|B) = \frac{P(B|A).P(A)}{P(B|A).P(A) + P(B|A^c).P(A^c)} = \frac{(.8)(.1)}{(.8)(.1) + (.2)(.9)} = \frac{8}{8+18} = \frac{4}{13}$$

Problem 41

Two patients share a hospital room for two days. Suppose that, on any given day, a person independently picks up an airborne infection with probability 1/4. An individual who is infected on the first day will certainly pass it to the other patient on the second day. Once contracted, the infection stays for at least two days. What is the probability that fewer than two patients have the infection by the end of the second day?

(A) 135/256

(B) 121/256

(C) 131/256

(D) 125/256

Problem 41 continued on next page. . .

B
If any patient gets infected on day 1, both of them will be infected on day 2. Let this be case A. Then P(A) = P(any one gets infected) = 1 - P(no one gets infected)

$$P(A) = 1 - \frac{3}{4}.\frac{3}{4} = \frac{7}{16}$$

Another way both could be found to be infected on day 2 is if neither of them contract the infection on day 1, but both of them get infected on day 2. Let us call this case B. Then

$$P(B) = \frac{3}{4}.\frac{3}{4}.\frac{1}{4}.\frac{1}{4} = \frac{9}{256}$$

Hence the total probability is $P(A) + P(B) = \frac{121}{256}$

Answer the next question using the following information.
Let $w = W/P$ be the real wage rate, where W is the nominal wage rate and P is the aggregate price level. The demand for labour is given by $D(w) = 1 - w$ and the supply of labour is described by the equation $S(w) = w$. If N is the employment level, then $f(N)$ is the aggregate output.

Problem 42

If nominal wage is always such that the labour market clears, then the aggregate supply curve is given by the equation

(A) $Y = Pf(N)$

(B) $Y = f(N)$

(C) $Y = Pf(1/2)$

(D) $Y = f(1/2)$

D
Labour market clears, so, $D(w) = S(w) \implies 1 - w = w \implies w = 1/2 \implies N = 1/2 \implies Y = f(1/2)$.

Answer the next five questions using the following information.
Consider the above described labour market with the following change: the nominal wage W minimizes $|D(W/P) - S(W/P)|$ subject to the constraint $W \geq W_0$, where W_0 is an exogenously given minimum nominal wage.

Problem 43

Given the price level P, the nominal wage W is

(A) $\max\{W_0, P/2\}$

(B) $\min\{W_0, P/2\}$

(C) $1/2$

(D) $W_0/2$

Problem 43 continued on next page. . .

A
$|D(W/P) - S(W/P)| = |(1-w) - w| - |1-2w|$ is minimized when $w = 1/2 \implies W/P = 1/2$. So the nominal wage would be $P/2$. But if thsi nominal wage is lower than the minimum wage W_0, then W_0 would be the prevalent nominal wage. Hence $W = \max\{W_0, P/2\}$.

Problem 44

Given the price level P, the employment level is

(A) $\min\{1/2, 1 - W_0/P\}$

(B) $\max\{1/2, 1 - W_0/P\}$

(C) $1 - W_0/2P$

(D) $1 - 1/2P$

A
Continuing from the previous problem, when even $W_0 < 1/2$, the market clears at $w = 1/2$ and then employment $= 1/2$. However, if $W_0 \geq 1/2$, then real wage would be higher ($w \geq 1/2$), hence demand for labour would be lower $(1 - W_0/P)$.

Problem 45

Given the price level P such that $1/2 \leq 1 - W_0/P$, the aggregate supply is

(A) $f(1/2)$

(B) $f(1 - 1/2P)$

(C) $f(1 - W_0/P)$

(D) $f(1 - W_0/2P)$

A
From the previous solution, we know that employment is $N = \min\{1/2, 1 - W_0/P\}$. Now since it is given that $1/2 \leq 1 - W_0/P$, $N = 1/2 \implies Y = f(1/2)$.

Problem 46

If the marginal productivity of capital is positive and the stock of capital increases, then the aggregate supply schedule will

(A) shift up

(B) shift down

(C) shift to the left

(D) shift to the right

D
Since capital stock increases, potential output of the economy would increase. Also at any given price level now more output would be produced. So the AS curve shifts to the right.

Problem 47

If the fixed nominal wage W_0 increases, then the aggregate supply schedule will

(A) shift up

(B) shift down

(C) shift to the left

(D) shift to the right

A
Since no additional capital stock is added, potential output would not change. So no rightward shift. But since W_0 increases, the firms would be willing to supply same output as before only if the prices are high as well. So the AS curve would shift up.

Problem 48

Consider a closed economy. If the nominal wage is flexible and nominal money supply is increased, then which of the following will be true in equilibrium?

(A) Real wage decreases and real money supply decreases

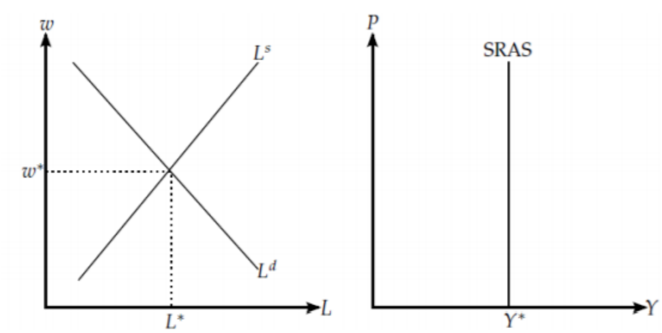
(B) Real wage decreases and real money supply increases

(C) Real wage is unchanged and real money supply is unchanged

(D) Real wage decreases and real money supply is unchanged

C

With flexible wages and prices, the labour market will adjust to a point where labour demand is equal to labour supply (there is an equilibrium real wage w = W/P). In the case where all agents have perfect information and do not suffer from money illusion, the equilibrium real wage and employment level will be independent of the price level P. With employment independent of P, the supply of output is also independent of P. This means the aggregate supply function is vertical.



Thus there will be a one-to-one increase in prices and nominal wages because of an increase in money supply. Thus the real wage and real money supply is unchanged.

Problem 49

Which of the following would make the LM curve flatter in the (Y, r) space?

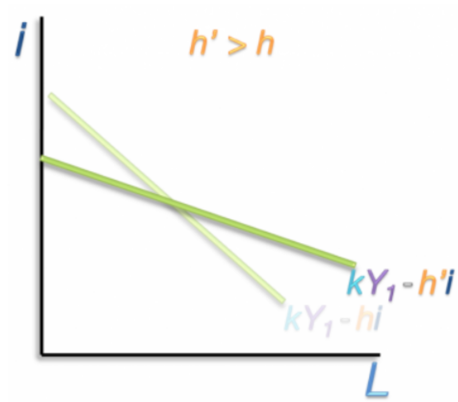
(A) An increase in income sensitivity of money demand

(B) An increase in interest sensitivity of planned investment

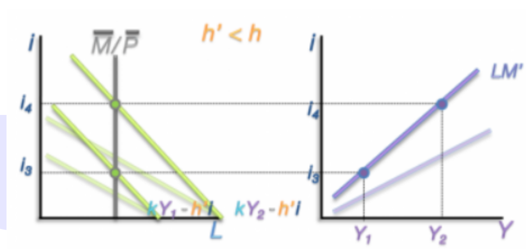
(C) An increase in the marginal propensity to consume

(D) An increase in the interest sensitivity of money demand

D
An increase in interest sensitivity of money demand makes the money demand curve flatter, as shown in the figure below



The corresponding effect on the LM curve would be that it becomes flatter as well. This is **reverse** of what is happening in the figure below



Problem 50

In an IS-LM model with fixed exchange rates and perfect capital mobility, an increase in government spending will lead to

(A) a deterioration in the trade balance

(B) an improvement in the trade balance

(C) no change in the trade balance

(D) an increase in export without affecting imports

Problem 50 continued on next page...

A
An expansionary fiscal policy tends to increase both interest rates and real GDP, so the capital account tends to improve while the current account tends to deteriorate.

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