Solutions: DSE entrance 2016

October 9, 2018

DSE Super 20

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Problem 1

If the following equation is estimated using OLS, and a 95% confidence interval for β_1 is constructed, then which of the following is true?

$$Y = \alpha + \beta_1 X_1 + \beta_2 X_2 + \varepsilon$$

(A) The OLS estimate of β_1 lies in the given interval with 95% probability

(B) The true value of β_1 lies in the given interval with 95% probability

(C) In repeated sampling 95% of the times the confidence interval will contain the true value of β_1

(D) Both (B) and (C)

C

Problem 2

What term would best describe the shape of the given boxplot?

(A) Right-skewed

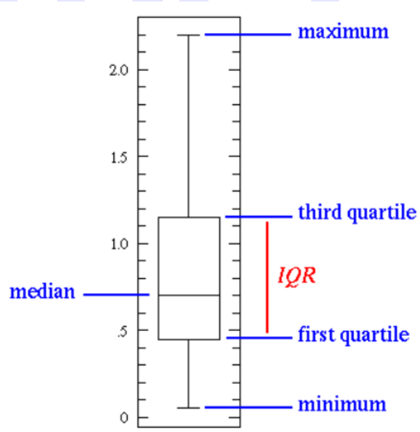
(B) Left-skewed

(C) Uniform

(D) Normal

A

A boxplot represents the datapoints as shown in the figure below:



Since in the given boxplot the median lies to the left of the midpoint, it will be a right skewed distribution.

Problem 3

The vitamin content of a particular brand of vitamin supplement pills is normally distributed with mean 490 mg and standard deviation 12 mg. What is the probability (approximately) that a randomly selected pill contains at least 500 mg of Vitamin C?

(A) 0.8

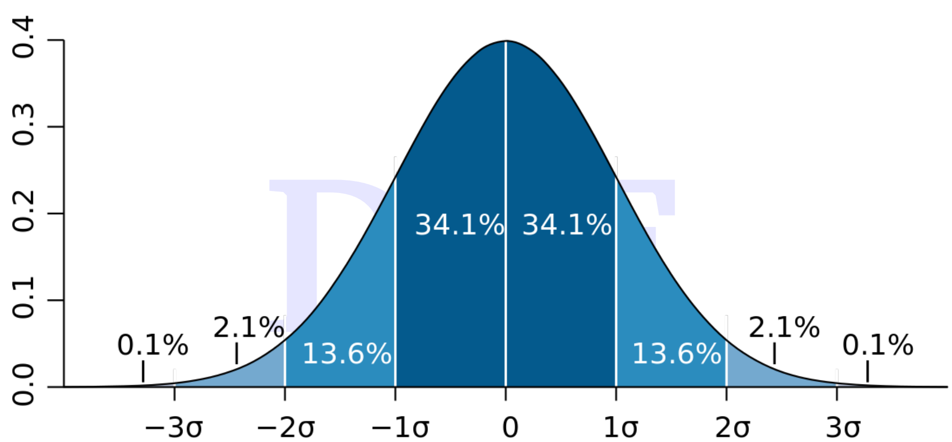
(B) 0.2

(C) 0.025

(D) 0.55

B

The probability contained within different standard deviations in a normal distribution is as shown below:



Since we are interested in probability of $C \geq 500$ we are roughly interested in the probability that the value is more than one σ greater than the mean. From the above plot that probability is approximately 0.2

Problem 4

A sample of 100 cows is drawn to estimate the mean weight of a large herd of cattle. If the standard deviation of the sample is 100 kg, what is the approximate maximum error in a 95% confidence interval estimate?

(A) 10

(B) 20

(C) 30

(D) 40

B

The 95% confidence interval is roughly $\bar{X}_n \pm 2\sigma'$. Now is the standard deviation $\sigma = 100$, the standard deviation of a sample of size 100 is $\sigma' = \dfrac{\sigma}{\sqrt{n}} = \dfrac{100}{\sqrt{100}} = 10$. Hence the maximum error would be $2\sigma' = 20$

Problem 5

Recent studies suggest that the migration to Indian cities from rural regions(MIG) can be explained be Quality of life(QL), state income as a ratio of Aggregate Indian income(Y) and the ratio of state employment to overall employment in India(E). Using data for the 29 Indian states, the following model is estimated:

$$\widehat{MIG} = -4.2 + 1.2QL - 0.6Y - 0.8E$$
$$(0.9) \quad (0.8) \quad (0.05) \quad (0.02)$$

The figures in the parentheses are standard errors. The t-statistic for the null hypothesis that the quality of life(QL) index does not impact migration is

(A) 0.8

(B) 0.9

(C) 1.5

(D) Insufficient information to calculate

C

$$t = \frac{|1.2 - 0|}{0.8} = \frac{3}{2} = 1.5$$

Problem 6

Consider the following estimated regression relating expenditures on Food(Y_i) to Income (X_i)

$$\hat{Y}_i = 145 + 0.3X_i - 0.1(X_i - X_i^*)D_i$$

where i denotes the individual; $X_i^* = 500$, this threshold distinguishes low-income from high-income individuals, and D_i takes value 1 if $X_i \geq X_i^*$ and 0 otherwise. All the estimated coefficients are significant at 5% level. Which of the following statements is *false* ?

(A) The marginal propensity to consume for people with low income is 0.3, and is lower for those with higher incomes; this makes sense as it is in accordance with Engel's Law

(B) This is a differential slope, common intercept dummy variable formulation with an additional restriction that leads to kinked Engel curve

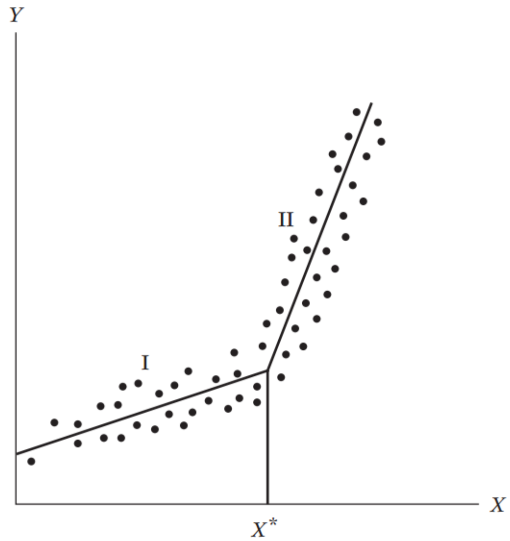
(C) This is a standard differential slope common-intercept dummy variable formulation

(D) Using this formulation yields predicted expenditure on food for people at income level 490 that is not very different from those people at income level 510 (estimate lie within 5 percent of each other)

Problem 6 continued on next page. . .

C

A standard differential slope common-intercept dummy variable formulation would lead to two lines of different slopes starting from the same intercept. This isn't the plot for the regression in consideration. Here we have a plot like below



Problem 7

The parameters of the following multiple regression model has been estimated using OLS.

$$Y = \beta_1 + \beta_2 X_2 + \beta_3 X_3 + \varepsilon$$

Denote the estimated residuals from this regression as e. Which of the following statements is *false*?

(A) The R-squared from the regression of e on a constant, X_2 and X_3 is zero

(B) The R-squared from the regression of Y on a constant and $\hat{Y}$ is the same as in the original regression ($\hat{Y}$ stands for estimated value of Y)

(C) The slope coefficient from the regression of Y on a constant and $\hat{Y}$ is 1 and the intercept is 0

(D) The R-squared from the regression of Y on a constant and e is same as in the original regression

D

A) e is that part of Y which could not be explained using X_2 and X_3. Hence the regression of e on X_2 and X_3 will be just the errors lending $R^2 = 0$. So this statement is **TRUE**.

B) $\hat{Y} = \hat{\beta}_1 + \hat{\beta}_2 X_2 + \hat{\beta}_3 X_3$. So the regression $Y = \alpha_1 + \alpha_2 \hat{Y} + u$ will be same as the regression $Y = \alpha_1 + \alpha_2[\hat{\beta}_1 + \hat{\beta}_2 X_2 + \hat{\beta}_3 X_3] + u$. The explained part in the last regression will be the same as before since we are effectively using the same variables. So this statement is **TRUE**.

C) The equation $Y = \alpha_1 + \alpha_2[\hat{\beta}_1 + \hat{\beta}_2 X_2 + \hat{\beta}_3 X_3] + u$, is same as $Y = (\alpha_1 + \alpha_2\hat{\beta}_1) + \alpha_2\hat{\beta}_2 X_2 + \alpha_2\hat{\beta}_3 X_3 + u$. Since the variables are same as before, the values of coefficients must also be the same as before. The values of α_1 and α_2 which give the same results as before is $\alpha_1 = 0$ and $\alpha_2 = 1$. So this statement is **TRUE**.

D) This is in contradiction to statement (A), and since (A) is true, this must be **FALSE**.

Problem 8

In the multiple regression model with 4 explanatory variables, with standard assumptions, estimated using ordinary least squares, all the coefficients turn out to be insignificant although the overall R-squared is high and associated F-statistic is significant. Also, pair-wise correlations amongst the four explanatory variables are all low, and range between 0.1 and 0.2, but are not statistically different from 0. Which of the following statements is *false*?

(A) Even though the pair-wise correlations between the explanatory variables are low, since they are (individually) statistically different from zero, OLS coefficients are likely biased

(B) This is likely a case of multicollinearity even though the pairwise correlations are low

(C) If the analyst is only interested in making forecasts then the insignificance of coefficients is per se not a problem since the F-statistic is significant and R-squared is high

(D) Dropping a variable may improve significance of remaining coefficients but they may be biased

A
This is clearly a case of multicollinearity. But the OLS estimators continue to be unbiased even when multicollinearity is present. So statement (A) is **FALSE**.

Problem 9

Instead of estimating a true cost function, which is described as a quadratic, where costs are regressed on an intercept, output and a quadratic term in output, a researcher estimates a linear function by regressing costs on an intercept and output. The estimates from linear cost function are likely to :

(A) have autocorrelated residuals

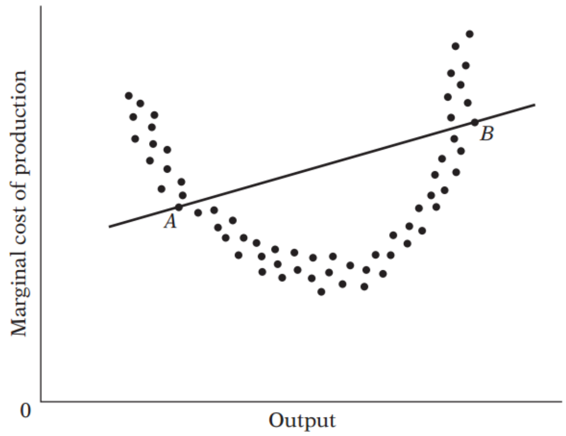
(B) be a biased estimate of marginal cost, even though there is no exact linear relationship between the linear and quadratic terms in output

(C) both (A) and (B)

(D) neither (A) nor (B)

C

We have committed a specification error here. The actual scatter plot and the regression line would be somewhat as shown below



As can be easily ascertained, there will be autocorrelation (refer Gujarati section 12.1). Also since we have committed a specification error, the OLS estimators are going to be biased.

Problem 10

Using data on class size (CS) and average test percentage(TP) from 101 classes, the following OLS regression is estimated:

$$\widehat{TP} = 96.4 - 1.12CS$$

R-squared is 0.1 and SER(Standard error of regression) is 5. What is the sample variance of test percentages across the 101 classes?

(A) 27.5

(B) 22.5

(C) 5.0

(D) 2.5

A

The relations between R^2, σ_{est} (SER) and σ_Y^2 (variance of Y) is

$$\sigma_{est} = \sqrt{\frac{(1-R^2)\sigma_Y^2}{df}}$$

Since sample size is $n = 101$, $df = n - 2 = 99$. Substituting the values in the equation we get

$$\sigma_Y^2 = 2750$$

Now sample variance of Y will be $\dfrac{\sigma_Y^2}{n-1} = 27.5$.

Problem 11

In the OLS regression $Y_i = \beta_1 + \beta_2 X_i + u_i$, suppose the coefficient of determination is estimated to be 0.6. We now transform the variables such that $Y_i^* = 0.5Y_i$ and $X_i^* = 0.75X_i$, and re-run the regression. The coefficient of determination is now:

(A) 0.6

(B) 0.4

(C) 0.9

(D) 0.3

A
Scaling does not affect the R^2. Hence it will remain unchanged.

Problem 12

The OLS regression of infant birth weight(BWT) on the mother's age(AGE) and years of mother's education(EDU) is:

$$\widehat{BWT} = \underset{(97)}{2600} + \underset{(3.5)}{2.3}AGE + \underset{(8)}{26}EDU$$

Where the standard errors are reported below the estimated coefficients. Sample size is 1000 and R-squared si 0.015. Sample information is provided in the table below:

	BWT	AGE	EDU
Mean	3000	25	10
Standard Deviation	500	5	2

A one standard deviation change in AGE is associated with an x standard deviation change in birth weight, where x is

(A) 0.046

(B) 0.007

(C) 0.023

(D) 0.035

C
We need to use the propoerty of standardized variables here, that is, if we run the regression

$$Y^* = \beta_1^* X^* + u^*$$

where all variables have been standardized, change in one standard deviation in the X^* value leads to a change in β_1^* standard deviations in the value of Y^*. Now using the property of standardized variables we can calculate $\beta_1^* = \beta_1 \dfrac{S_x}{S_y} = 2.3\dfrac{5}{500} = 0.023$. (Gujarati section 6.3).

Problem 13

If $f : \mathbb{R} \to \mathbb{R}$ is a continuous function ($\mathbb{R}$ is the set of real numbers) and E is an open subset of $\mathbb{R}$, then the set $\{x \in \mathbb{R} | f(x) \in E\}$ is necessarily

(A) open

(B) closed

(C) neither open, nor closed

(D) open and closed

A
A function $f : X \to Y$ is said to be continuous if the inverse image of every open set in Y is said to be open in X.

Problem 14

If X is a $n \times n$ non singular matrix, such that $XX^T = X^T X$ (X^T denotes the transpose of X). Let $Y = X^{-1}X^T$ where X^{-1} denotes the inverse of X. Then YY^T is equal to

(A) $I + Y$ (I is the identity matrix)

(B) I

(C) Y^{-1}

(D) Y^T

B

$$Y = X^{-1}X^T \implies Y^T = (X^{-1}X^T)^T = (X^T)^T.(X^{-1})^T = X.(X^T)^{-1}$$

$$YY^T = (X^{-1}X^T)(X(X^T)^{-1}) = X^{-1}(X^TX)(X^T)^{-1} = X^{-1}(XX^T)(X^T)^{-1} = (X^{-1}X)(X^T(X^T)^{-1}) = I$$

Problem 15

Suppose that $g(x)$ is a twice differential function and $g(1) = 1; g(2) = 4; g(3) = 9$. Which of the following is necessarily true? First and second derivatives of g are represented as g' and g'' respectively

(A) $g''(x) = 3$ for some $x \in [1, 2]$

(B) $g''(x) = 5$ for some $x \in [2, 3]$

(C) $g''(x) = 2$ for some $x \in [1, 3]$

(D) $g''(x) = 2$ for some $x \in [1.5, 2.5]$

C
Take $g(x) = x^2$. Then, $g''(x) = 2$. Now statements (A) and (B) are false for this functions, so they are not "generally true". (C) and (D) are related so that when ever (D) is true, (C) is true, but the converse is not true. It is possible that (C) is true but (D) is false. So (C) is the more general of the two statements, and hence we go with (C).

Problem 16

Consider $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = \sum_{i=1}^{11} |x - a_i|$ for all $x \in \mathbb{R}$, where $a_1 < \cdots < a_{11}$. This function has a minimum when x equals :

(A) a_2

(B) a_6

(C) a_7

(D) a_{11}

B

The Median Minimizes the Sum of Absolute Deviations. What we are basically trying to do is

$$\arg\min_x \sum_{i=1}^{N} |s_i - x|$$

One should notice that $\frac{\mathrm{d}|x|}{\mathrm{d}x} = sign\,(x)$ (Being more rigorous would say it is a Sub Gradient of the non smooth L_1 Norm function).
Hence, deriving the sum above yields $\sum_{i=1}^{N} sign\,(s_i - x)$.
This equals to zero only when the number of poisitve items equals the number of negative which happens when $x = median\,\{s_1, s_2, \cdots, s_N\}$.

Problem 17

Consider the set $\{x, y \in \mathbb{R}_+ \times \mathbb{R} | y \leq ln(x) - e^x\}$. This set is :

(A) a linear subspace of $\mathbb{R}^2$

(B) Convex

(C) convex and linear subspace of $\mathbb{R}^2$

(D) neither convex, nor linear subspace of $\mathbb{R}^2$

B

A linear subspace of X is such that if vectors $u, v \in X$, then $\lambda u + \gamma v \in X$ for all scalars λ and γ. Suppose (u_x, u_y) and (v_x, v_y) belong to the given set. Then $u_y \leq \ln(u_x) - e^{u_x}$ and $v_y \leq \ln(v_x) - e^{v_x}$.
Now if both λ and γ are negative, clearly $\lambda u_x + \gamma v_y$ will be negative and since negative values of x are not allowed in the set, it will not be in the set. Therefore the given set is not a linear subspace of $\mathbb{R}^{\ngeq}$
Now let $f(x) = \ln(x) - e^x$. Then the double derivative is $f''(x) = -\dfrac{1}{x^2} - e^x$ which is negative for all positive values of x (and the set is defined in such a way that x cannot take negative values). Hence the function is concave. Now a function is concave iff the lower epigraph is convex. So if $f(x)$ is concave, the lower epigraph $y \leq f(x)$ is convex.

Problem 18

A school with n students has m clubs to which they can belong, and students are allowed to be members of multiple clubs. It is known that

Problem 18 continued on next page. . .

(I) Each club has an odd number of members

(II) Every pair of clubs has an even number of common members (including 0)

Then it must be that:

(A) $m \geq n$

(B) $m \leq n$

(C) m and n can each be larger than the other, but $m \neq n$

(D) $m = n$

B
Assuming $m = 2$ and $n = 4$, one way to allocate memberships is to make one person each a member of just clubs 1 and 2, and make the remaining 2 students members of both the slubs. Such a membership pattern obeys the rules given. **Thus n can be greater than m.**
Assuming $m = n = 2$, onw way wo give memberships is to make one student a member of one club only. No student is a member of both the clubs here. Again the rules are obeyed. **So m can be equal to n.**
Now using the results above the only possible option is $n \geq m$

Problem 19

Let $n > 1$, and let S be the set of all $n \times n$ matrices whose entries are all chosen from the set {0,1}. Then the sum of the determinants of all these matrices

(A) is equal to 0

(B) is positive but less than $n^{\frac{n}{2}}$

(C) can be positive or negative, but is bounded by n^n

(D) is either -1 or 1

A
For any matrix A which can be formed from using just 1 and 0, the matrix A' which is obtained by doing one row exchange on A can also be formed form 1 and 0. Now these two matrices will have determinant values negative of each other. Hence summing up all such determinant values will yield 0.

Problem 20

$g : \mathbb{R} \rightarrow [0, 1]$ is a non-decreasing and right continuous step function such that $g(x) = 0$ for all $x \leq 0$ and $g(x) = 1$ for all $x \geq 1$. Let us define g^{-1} as follows, $g^{-1}(y) = \inf\{x \geq 0 | g(x) \geq y\}$. Which of the following is true about g^{-1} :

(A) is a continuous function

(B) is right-continuous but not left continuous

(C) is left-continuous but not right continuous

(D) neither left-continuous nor right-continuous

Problem 20 continued on next page. . .

C

The problem is wrong. The function is not defined for all points in the domain. Specifically not defined on $(0, 1)$. But still we are supposed to solve it.
Going ahead, for any $y \leq 0$, the set $\{x \geq 0 | g(x) \geq y\}$ will be the entire set $[0, 1]$. So the infimum of these values is 0.

For any value $0 < y \leq 1$, the set $\{x \geq 0 | g(x) \geq y\}$ will be the set $[1, \infty)$. This is because for all values of $x \geq 1$, $g(x) = 1$. so the infimum of these values is 1.

For all values fo $y > 1$, since $g(x) \geq 1$ is not possible, the set itself is a null set. Hence the set has no infimum. So the final function $g^{-1}(y)$ is left continuous.

The next two questions pertain to the following:
Consider an exchange economy with two agents, 1 and 2, and two goods, X and Y. There are 6 units of X and 4 units of Y available. Agent 1 has the utility function $u_1 = min\{x_1, y_1\}$ and agent 2 has the utility function $u_2 = x_2 + y_2$.

Problem 21

Which of the following allocations is not Pareto efficient?

(A) (2,0), (4,4)

(B) (4,4), (2,0)

(C) (2,2), (4,2)

(D) (0,0), (6,4)

A

All points on the line $X_1 = y_1$ on the edgeworth box are pareto optimal. Only option (A) does not lie on this line.

Problem 22

The set of equilibrium prices for this exchange economy is given by?

(A) $\{p_x > 0, p_y > 0, \frac{p_x}{p_y} \leq 1\}$

(B) $\{p_x > 0, p_y > 0, \frac{p_x}{p_y} = 1\}$

(C) $\{p_x > 0, p_y > 0, \frac{p_x}{p_y} \geq 1\}$

(D) Insufficient information to conclude

A

For a point like $(5, 4))$, the agent 1 is willing to give up his one unit of good 1 free of cost. But for most cases the equilibrium prices are going to be equal to 1. Hence the set of equilibrium price vectors is $\frac{p_x}{p_y} \leq 1$.

Problem 23

The market for widgets has a demand function $Q = 100 - 10p$, where Q is the quantity demanded and P is the price in rupees. There are 10 price taking firms in the market, each having a cost function $c(q) = \frac{1}{2}q^2$, where q is the firm's own output. There is no new entry. Suppose the government imposes an excise tax of Rs. 2 per unit of widgets, to be paid by sellers, the equilibrium market price is:

(A) 7

(B) 6

(C) 5

(D) 2

B
Let P_c be the price that consumer pay per unit. Then the amount sellers get after tax is $P_s = P_c - 2$. The marginal cost for each firm is $MC(q) = q$. Hence the supply function of a firm is $P_s = q$. So the supply function of the market is $Q = 10q = 10P_s = 10(P_c - 2)$. Also the demand function is $Q = 100 - 10P_c$. Solving the supply and demand equations we get $P_c = 6$.

Problem 24

A consumer has utility function $u\{x_1, x_2\} = min\{3x_1 + x_2, x_1 + 2x_2\}$. Prices of the two goods are p_1 and p_2 respectively. The consumer will buy positive quantities of both the goods if and only if the price ratio p_1/p_2 is"

(A) greater than 3

(B) between 1/2 and 3

(C) between 2 and 3

(D) less than 1/2

B

Problem 25

Utility function of a consumer over three goods X, Y and Z is $U = y.min\{x, z\}$. Prices of all the three goods are the same in the market. Three discount deals are available, which are as follows:
Deal I: Get a unit of Z free with a unit of X
Deal II: Get a unit of Z free with a unit of Y
Deal III: Get 1/2 unit of X and 1/2 unit of Z free with a unit of Y
Which of the deals should the consumer choose?

(A) Deal I

(B) Deal II

(C) Deal III

(D) All the deals are equally good

Problem 25 continued on next page. . .

C
Assume all goods are numeraire.
Deal I: The consumer never buys any z separately because instead spending that amount on buying X would give the same amount of both y and x. So the utility function becomes $U)1 = y.\min\{x, x\} = yx$.
Deal II: The consumer never buys z separately because at the same price he can get both a unit of y and a unit of z. So the utility function becomes $U_2 = y \min\{x, y\}$. From here we can deduce that the consumer will always buy at $x = y$. So the function becomes $U = xy$.
Deal III: The consumer neve buy x and y separately, because he owuld then need to buy them in equal quantities. Instead of buying a unit of each in 2 rupees, he can get a unit of each free with 2 units of y in the same cost. So the utility function becomes $U_3 = y \min\{\frac{y}{2}, \frac{y}{2}\} = \frac{y^2}{2}$.
with the assumed prices one can check that deal 3 is the best.

Problem 26

Two widget producers, A and B, operate in perfectly competitive input and output markets. Firm A uses capital (k) and labour (l) to produce widget; its production function is $f_1(k_1, l_1) = (k_1 l_1)^{1/3}$. Firm B uses only labour; its production function is $f_2(l_2) = \sqrt{l_2}/(1 + k_1)$. Efficiency of input allocation can be improved by:

(A) imposing a tax on capital use

(B) merging firms A and B

(C) providing a subsidy on labour use to firm A

(D) all of the above

D
Under a more efficient plan, less of k_1 should be used. This could be done by either taxing capital, subsidising labour, or even merging the firms.

The next two questions pertain to the following:
A student has the opportunity to take a test at most thrice. The student knows that each time she takes the test, her score is an independent random draw from the uniform distribution [0,100]. Each time the student takes the test and learns her score, she can either stop and accept it as her official score, or she can discard the result and retake the test. However, after the third attempt, the student has no more opportunities to retake the test. In that case,her score on the last(i.e., third) try will be her official score. The student's objective is to maximize her expected official score.

Problem 27

If the student follows an optimal plan, her final expected score before taking any of the tests is approximately:

(A) 30

(B) 50

(C) 70

(D) 90

C
The student has an expected marks of 50 from taking the test any time. So she will reject the test 2 if $M_2 < 50$ So the expected marks before taking second test is

$$E(T_2) = \frac{50}{100}(50) + \int_{50}^{100} \frac{1}{100} M_2 dM_2 = 25 + \frac{10000 - 2500}{200} = 62.5$$

So the student has an expected marks of 62.5 before taking the second test. SO she will reject the first test if $M_1 < 62.5$. Thus the expected marks before taking the first test is

$$E(T_1) = \frac{62.5}{100}(62.5) + \int_{62.5}^{100} \frac{1}{100} M_1 dM_1 \approx 39 + 30 \approx 70$$

Problem 28

Now consider the case when the university allows the student to take the final test only when her score is below 40 in the second test. The student retains the choice to stop or retake after the first attempt. The student will decide to be retested after the first test if and only if her score is less than:

(A) 37.5

(B) 50

(C) 62

(D) 67.5

C
The expected score after second test is now

$$E(T_2) = \frac{4}{10}(50) + \int_{40}^{100} M_2 \frac{1}{100} dM_2 = 20 + \frac{10000 - 1600}{200} = 62$$

The next two questions pertain to the following:
Consider a homogeneous goods market with two firms. Let x_1 and x_2 be the quantity choices of Firms 1 and 2 respectively. The total quantity is $X = x_1 + x_2$. The inverse demand function is $P = a - bX$, where P is the market price and A and B are the intercept and slope parameters respectively. Both firms have the same marginal costs denoted by c. Suppose $b > 0$, and $0 < 3c < a$.

Problem 29

Suppose firm 1 has an objective of maximizing revenue and firm 2 has an objective of maximizing profit. Both firms choose quantities simultaneously. Then:

(A) Firm 1 has larger market share and larger profits

(B) Firm 2 has smaller market share but larger profits

(C) Firm 1 has smaller market share but larger profits

(D) Firm 2 has larger market share and larger profits

C
A profit maximizing firm solves MR=MC whereas a revenue maximizing firm solves MR=0. Since MC is given to be greater than zero, revenue maximizing firm would produce more quantity. However since both the firms are in cournot competition, the price would be the same. And given that even marginal costs are same, the firm producing more would get more profits.

Problem 30

Suppose both firms become revenue maximizers. Then:

(A) Both will produce more than Cournot output

(B) Both will produce less than Cournot output

(C) Both will produce the perfectly competitive output

(D) Both will produce more than the perfectly competitive output.

A

The next two questions pertain to the following:
Four cities A,B,C,D are located as vertices of a square ABCD, and are connected by roads that form the four sides of the square. Mr. Rand Walker travels thus: if he is at city $i (i \epsilon \{A, B, C, D\})$ in period t, then he randomly, with probability 1/2 each, moves to one of the two vertices/cities that are adjacent to i in period $t + 1$.

Problem 31

If Mr. Walker is at city A at time $t = 0$ then the respective probabilities with which he is at cities A,B,C,D in period $t = 10$ are:

(A) 1/4, 1/4, 1/4, 1/4

(B) 0, 1/2, 0, 1/2

(C) 1/2, 0, 0, 1/2

(D) 1/2, 0, 1/2, 0

D
In every odd period he moves to one of cities B and D, and in ever odd period he is in one of cities A and C. Since the probabilities are symmetric, in an even period like $t = 10$, he is equally likely to be in either A or C.

Problem 32

If Mr. Walker is at city A at time $t = 0$, then what is the probability that he never visits city A again till(including) period $t = 10$?

(A) $(1/2)^5$

(B) $(1/2)^{10}$

Problem 32 continued on next page...

(C) $(3/4)^5$

(D) $(3/4)^{10}$

A
Starting from A, he will move to either B or D in $t = 1$. Then in every future even period he needs to make a choice of moving to either of A or C. In any period he refuses to go to A with a probability of 1/2. So refusing to go to A in each of the 5 even periods 2,4,6,8 and 10 would happen with a probability of $(1/2)^5$.

Problem 33

Consider a railway signalling system: a signal is received by the station A from the traffic control office and then transmit it to station B. Suppose that at the origin (traffic control office) signal can be yellow or red with probability 4/5 and 1/5 respectively. The probability of each station receiving the signal correctly from its predecessor is 3/4. If the signal received at station B is yellow, then the probability that the signal was yellow is

(A) 22/23

(B) 11/20

(C) 20/23

(D) 9/20

Using Bayes' theorem

$$P = \frac{(4/5)(3/4)}{(4/5)(3/4) + (1/5)(1/4)} = \frac{12}{13}$$

Problem 34

Suppose X has a normal distribution with mean 0 and variance σ^2. Let Y be an independent random variable taking values -1 and 1 with equal probability. Define Z=XY + X/Y. Which of the following is true?

(A) Var(Z)> σ^2

(B) Var(Z)< σ^2

(C) Var(Z)=σ^2

(D) Var(Z) can be greater than or smaller than σ^2

A
Since Y takes values of only 1 and -1, we have $XY = X/Y \implies Z = 2XY$. So now

$$var(Z) = var(2XY) = 4var(XY) = 4[E(XY)^2 - (E(XY))^2]$$

Since X and Y both have expectations zero and they are independent, $E(XY) = 0$. Also

$$E(XY)^2 = E(X^2).E(Y^2) = E(X - \mu_X)^2.E(Y - \mu_Y)^2 = \sigma_X^2.\sigma_Y^2$$

Now we can easily calculate that $\sigma_Y^2 = 1$ Thus we finally get

$$var(Z) = 4\sigma^2 > \sigma^2$$

Problem 35

According to Ricardian equivalence proposition, a reduction in the current (lump sum) taxation of household income

(A) would reduce current consumption, but leave future consumption unaffected

(B) would reduce future consumption, but leave current consumption unaffected

(C) would reduce both current and future consumption

(D) would leave both current and future consumption unaffected

D
Ricardian equivalence states that if the government reduces taxes, people xpect higher taxes in the future and thus start saving more in the current period. So the combined effect of an increase in disposable income and reduction in MPC leads to an overall same amount of consumption, in both the current and future periods.

Problem 36

Tobin's q theory suggests that firms will find it profitable to invest when the value of Tobin's q is:

(A) greater than zero

(B) less than zero

(C) greater than unity

(D) less than unity

C

The next 4 questions pertain to the following:
Consider an agrarian economy with two single membered households. The households are engaged in own cultivation using their family land, labour and capital. Each household is endowed with 1 acre of land and 1 unit of labour. However the two households differ in terms of their initial capital endowment (K_0^R and K_0^P), where R denotes the relatively richer household and P denotes the relatively poorer household. Assume $2 < K_0^R < 4$ and $0 < K_0^P < 1$. The household have access to two technologies which are specified by the following production functions:

$$\text{Technology A: } Y_t = (N_t L_t)^{1/2}(K_T)^2;$$
$$\text{Technology B: } Y_t = (N_t L_t)^{1/2}(K_T)^{1/2}$$

where N_t represents land(in acres), L_t represents labour, and K_t represents capital respectively.
The households choose the technology which gives them higher output (given their land, labour and capital stock) in any period t. In every period they consume half of their total income and save the rest, which adds to the next period's capital stock. Land and labour stock remain constant over time. Existing capital stock depreciates fully upon production.

Problem 37

Given their initial factor endowments, the technology choices of the rich and the poor households respectively are:

(A) A,B

(B) B,A

(C) A,A

(D) B,B

A
Given that both households have $L_t = N_t = 1$, $2 < K_0^R < 4$ and $0 < K_0^P < 1$, the richer household would be better off under tech A and the poorer one under tech B.

Problem 38

In the short run, the average capital stock in the economy $(\overline{K})$ evolves according to the following dynamic path:

(A) $\dfrac{d\overline{K}}{dt} = \dfrac{1}{4}[(K_t^R)^{1/2} + (K_t^P)^2 - 2(K_t^R + K_t^P)]$

(B) $\dfrac{d\overline{K}}{dt} = \dfrac{1}{4}[(K_t^R)^2 + (K_t^P)^2 - 2(K_t^R + K_t^P)]$

(C) $\dfrac{d\overline{K}}{dt} = \dfrac{1}{4}[(K_t^R)^{1/2} + (K_t^P)^{1/2} - 2(K_t^R + K_t^P)]$

(D) $\dfrac{d\overline{K}}{dt} = \dfrac{1}{4}[(K_t^R)^2 + (K_t^P)^{1/2} - 2(K_t^R + K_t^P)]$

D
Since the rich household uses tech A, $Y_R = (K_t^R)^2$. Similarly $Y_P = (K_t^P)^{1/2}$. Now since the households save half the income and previous capital stok completely depreciates, we have $\dfrac{d\overline{K}}{dt} = \dfrac{1}{4}[(K_t^R)^2 + (K_t^P)^{1/2} - 2(K_t^R + K_t^P)]$.

Problem 39

In the long run:

(A) income of both households grow perpetually

(B) income of household R grows perpetually but that of household P approaches a constant

(C) income of household P grows perpetually but that of household R approaches a constant

(D) income of household R grows perpetually but that of household P falls perpetually

Problem 39 continued on next page...

B
For the Rich household, $K_{t+1} = \frac{1}{2}K_t^2$. Now this value would be greater than K_t when

$$\frac{1}{2}K_t^2 > K_t \implies K_t > 2$$

Since it is given that the rich household begins with $K_0 > 2$ it will keep on getting richer for ever.

For the poor household, $K_{t+1} = \frac{1}{2}\sqrt{K_t}$. Now this value will increase if

$$\frac{1}{2}\sqrt{K_t} > K_t \implies K_t < \frac{1}{4}$$

So the poor household will witness an increase in capital if the initial capital stock is lower than 1/4, and witness a reduction in capital stock if the initial capital is higher than 1/4. In either case, it will reach the constant of $K_p = 1/4$ over the long run.

Problem 40

If, at the end of the initial time period, the households were given a choice to spend their savings in buying more land instead of investing in capital stock:

(A) both households would have bought more land

(B) both households would have still invested in capital

(C) Household R would have still invested in capital but household P would have bought more land

(D) Household P would have still invested in capital but household R would have bought more land

B
Since the entire capital stock from previous period depreciates, andoutput at zero capital is zero for either of the production functions, both households will have to invest in capital instead of land.

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